

Energy Yield of a School PV-System

1. Introduction

a. Learning Objectives

In the summer of 2011, a photovoltaic system was installed on our school property by the public utility company. The solar cell module was attached to a pole and could track the Sun. The energy generated is fed into the power grid via inverters. The data-output of the plant could be retrieved by colleagues on the Internet.

The interpretation of the diagrams as well as the evaluation of the data by working with a table is exemplified over the period of a day. The example is also suitable as a practical introduction to integral calculus.

b. Scientific Background

The key physical point is the generation of electrical energy from solar energy, whereby the focus here is not on the processes in the solar cells, but on maximizing the energy yield. The solar module is designed in such a way that it can always be optimally aligned to the Sun's rays by rotating and tilting. Thus, one obtains an energy yield optimized by technology.

The values thus provided by the system are evaluated using mathematical methods. This way of dealing with tabular data forms the mathematical core of the exercise.

c. Connection with Sustainability Development Goals

This is mainly about Goal 7, clean and affordable energy.

2. Background and Exercise

In the summer of 2011, a photovoltaic system was erected on our school property. The solar cell module was attached to a pole and could track the Sun (rotated and tilted) (Fig. 1). It had an area of 51 m² and a peak power of 7.35 kW. The system was set up in such a way that only in the winter months of December and January was there a slight shading from a part of the building. The energy generated was fed into the power grid via inverters for remuneration. However, this yield did not benefit the school, as the Bad Pymont public utility company had the system erected and also operated it. In the meantime, the plant has been moved again to allow for new construction in the school.

Since January 2012, the data of the plant could be accessed by colleagues on the Internet, where they were presented very clearly (Fig. 2).



Fig. 1

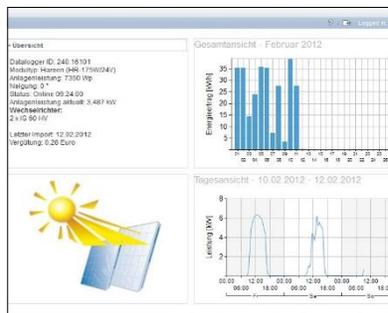


Fig. 2

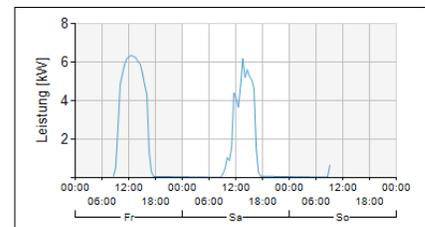


Fig. 3

Figure 3 shows the time course of the electrical power for 2 weekdays (Friday, February 10, 2012, and Saturday, February 11, 2012). On Saturday, the sky was temporarily slightly cloudy; on Friday, however, the sun shone undisturbed the whole day. I shall now examine this day in more detail.

The system transmitted the current value of the power every half hour and created from it the diagram P(t), a polygonal curve, from which one could read the measured values (fig. 4 and table 1). From the bar chart in the upper right corner of Fig. 2 it can be seen that on Friday about 40 kWh of electrical energy E was fed into the power grid. The exact value is 39.29 kWh.

How exact can this value be determined from the available data? The display for one day can be changed so that the measured values can be read and transferred to a table. The curve of the graph is very nice and smooth with a small disturbance in the right part - this is the shading mentioned above (Fig. 4).

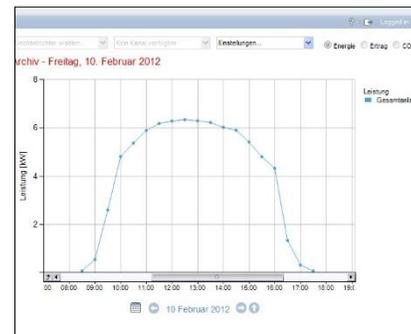


Fig. 4

Zeit	8.00	8.30	9.00	9.30	10.00	10.30	11.00	11.30	12.00	12.30	13.00
P/kW	0,00	0,08	0,55	2,60	4,82	5,38	5,90	6,19	6,29	6,35	6,30
Zeit	13.30	14.00	14.30	15.00	15.30	16.00	16.30	17.00	17.30	18.00	
P/kW	6,23	6,03	5,91	5,42	4,81	4,33	1,34	0,32	0,08	0,00	

Table 1

For the power P during a time period Δt applies: $P = \frac{\Delta E}{\Delta t}$. The total energy E is obtained by summing: $E = \sum \Delta E = \sum P \cdot \Delta t$. Thus, E is given by the area under the graph of power. The power (also called energy flux) is thus the rate of change of energy transferred. Thus, the evaluation of the data is a realistic, practical example for the introduction of the integral calculus.

Several approaches are possible for the area calculation. For this, the data must be transferred to *Lists&Spreadsheet* (Fig. 5, columns t and p). In figure 6 the scatter plot (connected thick points) was created with it. For the calculations it is useful to insert another column p2, in which the data from p are shifted up by 1 by $shift(p, 1)$ (Fig. 7, and already shown in fig 5.).

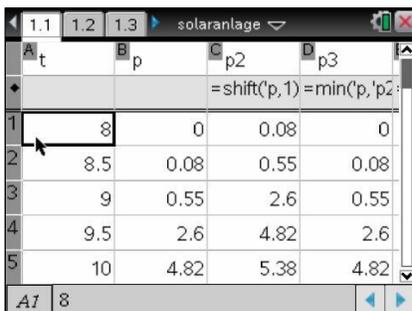


Fig. 5

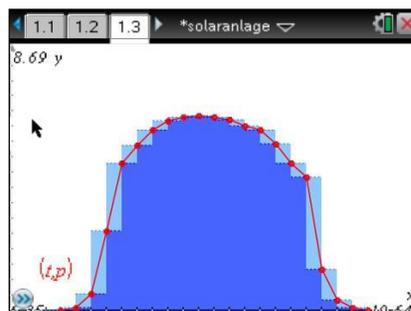


Fig. 6

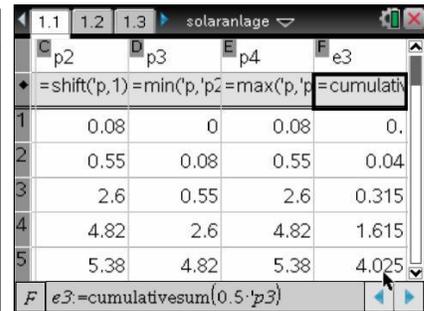


Fig. 7

For the lower sum (dark blue area in Fig. 6) the minimum of p and $p2$ is stored line by line in $p3$ ($min('p, 'p2)$). In $e3$ the individual rectangular areas with the area $0.5 \cdot p3$ are calculated and summed up to the total area (total energy) ($cumulativsum(0.5 \cdot p3)$). For the upper sum (light plus dark area in Fig. 6), the maximum from p and $p2$ is calculated in $p4$ and the total energy in $e4$. One obtains $E_U = 36.29$ kWh and $E_O = 42.64$ kWh, as expected very poor approximations for the actual value $E = 39.29$ kWh.

A better approximation can be expected if one calculates the area under the polygonal train. It is composed of trapezoids, which can be easily calculated with the data already generated by taking the average of $e3$ and $e4$ in another column $e5$. This gives the approximation $E_T = 39.465$ kWh. Since the deviation from the actual value is only about 0.5%, this is a very good approximation, which cannot be further improved with the available data. The small deviation can probably be explained by the fact that in reality Δt should be much smaller than 30 minutes.