Heads and Tails

Teacher Notes

Introduction

This activity, which has been specially written for use with TI-Nspire Navigator, is about experimental probability. Students are able use their TI-Nspire handhelds to record the results when they toss one, two or more coins. Each handheld immediately displays a bar chart of the entered results and these can all be collected by the teacher for display simultaneously on a large screen, allowing individual variations to be discussed. The results from the whole class can then be compiled and displayed on a single bar chart.

The key mathematical point is that with small samples there is a lot of variability around the theoretical probability, but with a large sample that variability is likely to be much less. The activity may also be used as an introduction to the binomial theorem

Resources

There are two TI-Nspire documents – one for distribution to students' handhelds and one for use by the teacher.

Skills required

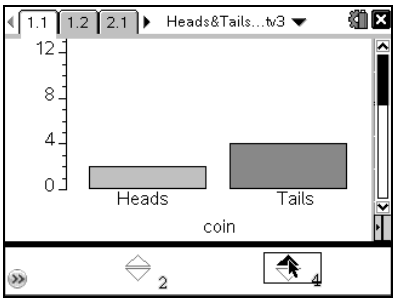
Students will need basic familiarity with using the handhelds. In particular they will need to:

- open the TI-Nspire document;
- operate the slider by clicking appropriately;
- move to a new page of the document;
- grab and drag points on the screen.

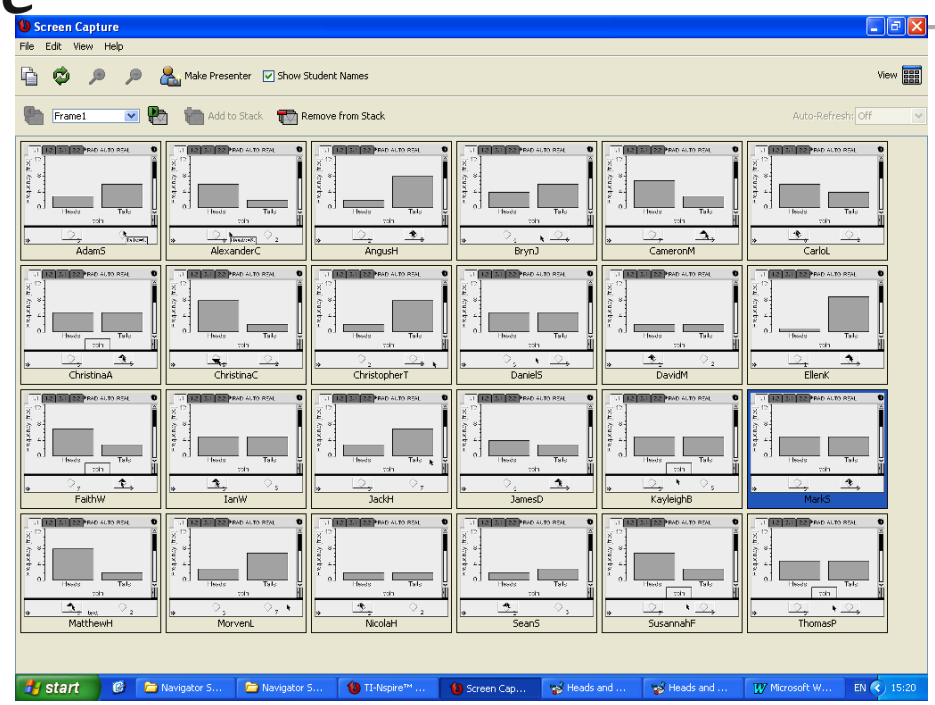
The activity

Toss a single coin

Once Navigator is installed and all handhelds are connected the student version of the tns file should be transferred to all the handhelds.
Students open the document and begin tossing a coin and recording their results on page 1.1. This can be done by clicking on the sliders as shown here.



As the results are entered, a bar chart is displayed on each handheld screen and, by means of the Screen Capture tool, these can be transferred to the teacher's computer for display on the large screen at the front of the class. The teacher may ask for a small sample of roughly 10 tosses of the coin to be recorded in the first instance. "What do you expect the outcome to be? an equal number of heads as tails?… always?... mainly? Let's try it and see."



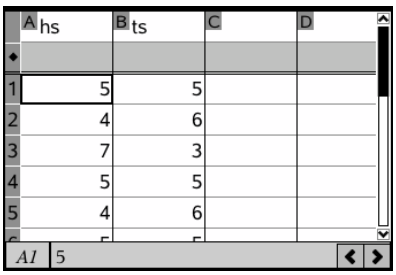
It quickly becomes clear that, although equal numbers of heads and tails is a likely outcome, there is a wide variation and this can be discussed. With small samples there is considerable variation around the most likely outcome.

But what would the distribution look like if all the class results were pooled?

The teacher version of the TI-Nspire document can be loaded onto the Teacher Edition of the software. If individual student results are entered on the spreadsheet (page 1.1) the corresponding bar chart for the whole class appears on page 1.2.

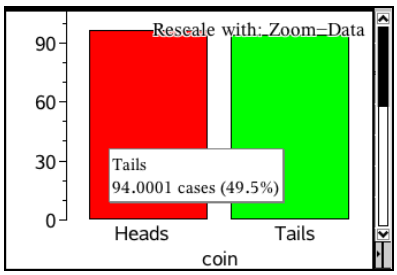
(Note: It is quicker to collect all the heads data first – pressing Enter to move to the next line, then repeat the process for the tails data. It isn't important if the heads and tails data don't match as only the totals are used on page 1.2).

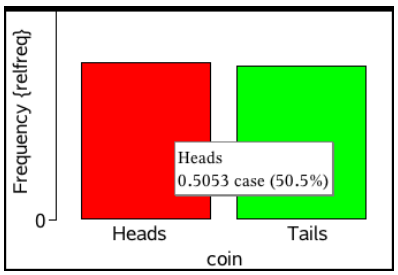
The vertical axis can be re-scaled using the Window Menu, option 2: Zoom – Data, or simply by dragging a point on the axis
Page 1.3 provides the corresponding bar chart showing relative frequency. Moving the cursor to point at the top of a bar will display the relative frequency, which is likely to be close to 0.5. Despite the disparity in the individual small samples, when they are collected together the distribution closely resembles what theory would predict.



	A hs	B ts	C	D
1	5	5		
2	4	6		
3	7	3		
4	5	5		
5	4	6		

A1 5

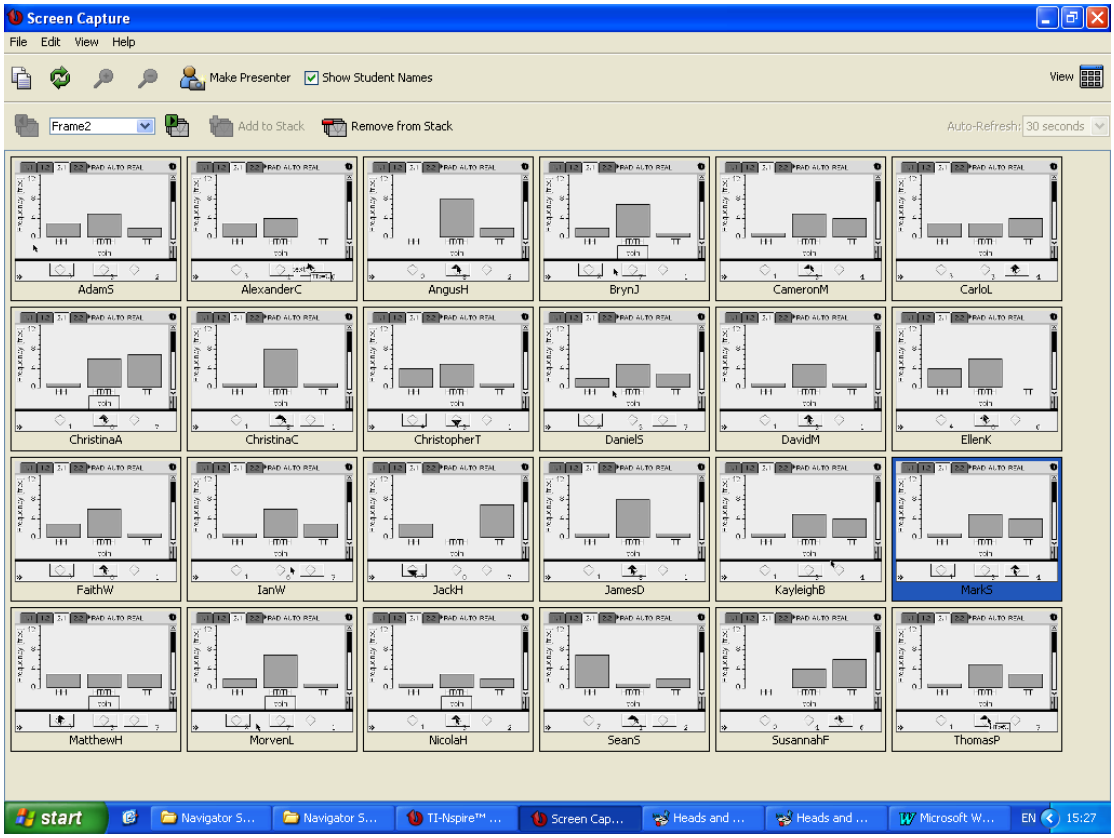




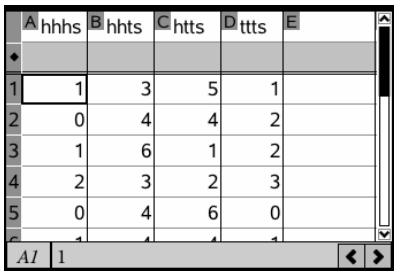
NB If the teacher is using TI-Nspire software viewed in Normal mode, coloured bars are shown.

Two and three coins

Using Problems 2 and 3 in both the student and teacher versions of the TI-Nspire document, the procedure can be repeated for the situations where students toss two and three dice.

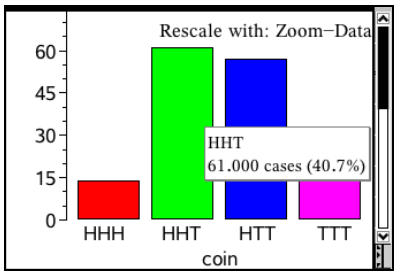


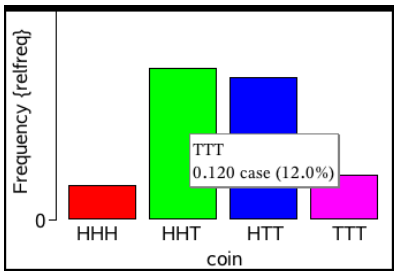
When compiling the class results it is possible to draw attention to the relative frequency graphs where experimental probabilities are plotted. Why are they the shape that they are? What do we notice about the values of the relative frequencies? Why to they add up to 1?



	A hhhs	B hhts	C htts	D ttts	E
1	1	3	5	1	
2	0	4	4	2	
3	1	6	1	2	
4	2	3	2	3	
5	0	4	6	0	

A1 1

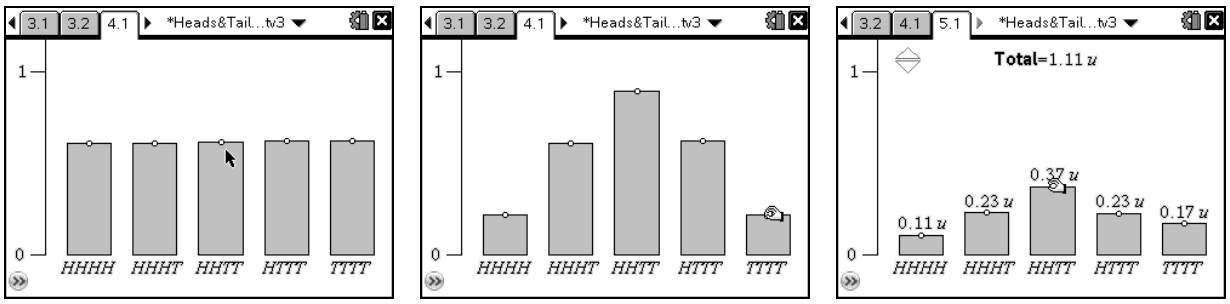




Predictions for 4 coins

Having gained insights into the likely outcomes for one, two and three coins students can no be challenged to make a prediction for the outcome with four and five coins. On page 4.1 students are presented with a mock-up graph and asked to adjust it to show what they think it will look like if there are four coins being tossed. To do this they can grab and drag the open circles at the top of each bar.

It is one thing to get an appropriately shaped graph and another to make certain that the relative frequencies have a sum equal to 1. Page 5.1 provides a similar graph but this time with the relative frequencies (or experimental probabilities) displayed. You might like to encourage students to explore this before encouraging them to think about what the sum of the probabilities must be. Clicking on the slider at the top of the page will also cause the total of all the relative frequencies to be displayed. Students then need to drag the bars so that the total equals 1.



Once students are under way with this activity it is again instructive to use Screen Capture on Navigator in order to see and discuss different approaches.