Diameters of Polygons
Teacher Notes

Introduction

It's not only circles that have diameters: the OED defines a diameter as "a straight line passing from side to side through the centre of a body or figure, *especially* a circle or sphere". In this activity various regular polygons are drawn and the ratios of their perimeters to their diameters calculated. These ratios are then investigated for increasing values of n, the number of sides of the polygon.

I decided to carry out this activity with two middle ability Year 8 classes in 2009 and the following is an account of the three lessons that it occupied. The account includes full instructions and key presses needed to carry out the simple TI-Nspire processes.

I had an idea where it was going, but using the TI-nspires took me down some new pathways to get there and although we experienced some stony ground on the way the lush pastures at the end well repaid the journey!

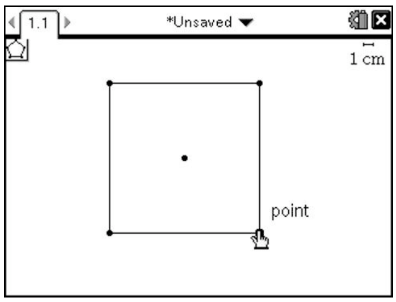
Resources

There is no pre-structured tns file for this activity: we started with a new blank TI-Nspire document. Students were guided through the work using the Nspires to construct their own files and explore the menus themselves.

The first lesson

The objective of the first lesson was to get students used to working with the menus on the geometry page and to investigate the ratio of the perimeter to the diagonal (or diameter) of a square. Future lessons would look at similar relationships between the perimeter and the diameter of regular polygons, looking first at those with an even number of sides then those with an odd number of sides. Graphs would be drawn of the results and we would look at what happens to the ratios as the number of sides gets larger.

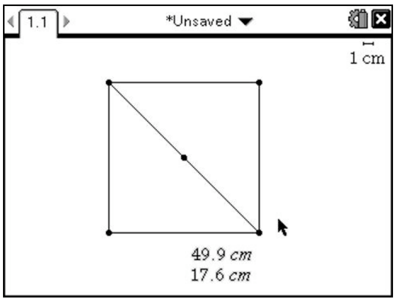
We started by opening a new document and selecting a Geometry page.



We then drew a square in the centre of the screen using the regular polygon tool: (menu)(9)(5) followed by (enter) to fix the centre, dragging the pencil out, and then (enter) to fix the radius of the circumscribed circle. We then rotated the cursor clockwise until a square was obtained, and pressed (enter) again.

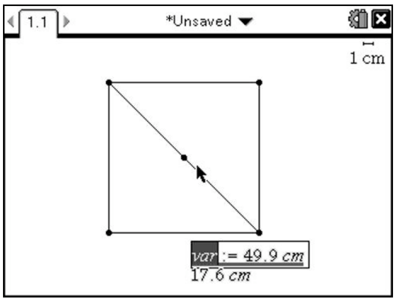
Firstly I demonstrated this to the class using the TI-Nspire Teacher edition of the software on the Interactive Whiteboard (IWB). Then the students did it for themselves on their handhelds. Experience has taught me that they need to watch the whole procedure first before trying it themselves. Then those who have not remembered are helped by those who do and this helps everyone make good progress.

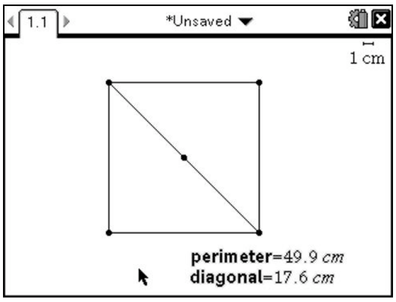
The next step was to draw a diagonal, using menu 7 5. Word soon got round the classroom that the cursor changed when moved over one vertex of the square and that was the time to press [click], or enter. The next step was to measure the perimeter and length of the diagonal using menu 8 1, clicking on the square, then moving the cursor to where we wanted to display the measurement, then pressing enter again.



We then named these variables so that we could capture the data in a spreadsheet. I used the names *perimeter* and *diagonal*, but in a class of 30 students there was more variation! This caused a small hiccup with some later when they used a different spelling of their variable when attempting to capture the data, but this was soon sorted.

To name the variable, select the pointer (menu 1 1), click on the measurement to highlight it, then press var 1 and type in the name you want. Finally press enter to identify that variable.

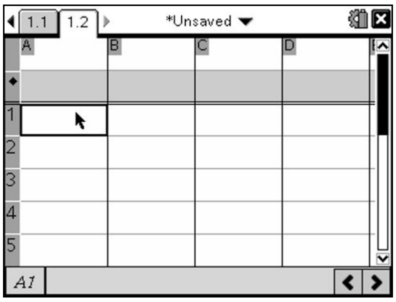




You can then move these measurements around to place them where they are easily visible by moving the cursor over the measurement, holding down [click] for a good second until the hand closes, then moving the cursor until the measurement is where you want it. Press [click] again to drop it.

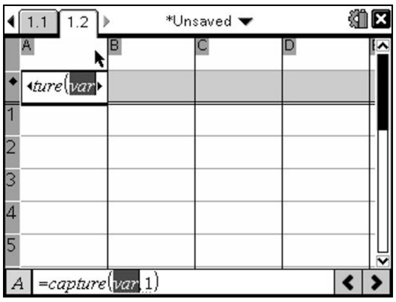
Students then grabbed a vertex and moved it so that they could see the changes in the square and measurements.

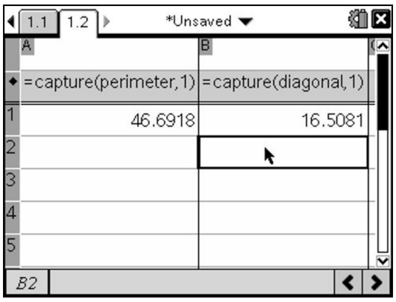
Some noticed that the perimeter was always just less than three times the diagonal, so we opened a new page to collect the data and investigate the connection: ctrl doc 4 inserts a new Lists & Spreadsheet page.



To capture the data, move the cursor up into the grey cell and press ctrl menu 8 1. The following screen appears.

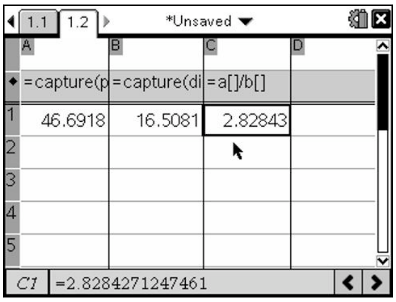
Now type *perimeter* and press enter. Repeat with *diagonal* in the next column:



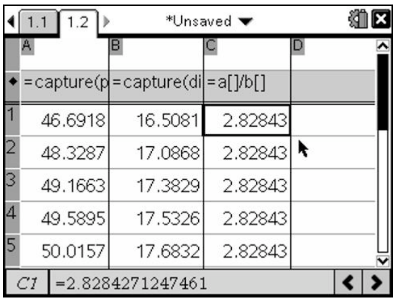


Note: the columns have been expanded here so that the full headings can be seen.

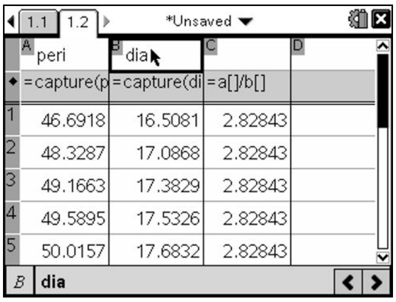
Students now moved to the next column and typed *=a/b* (enter). The handheld does the rest.



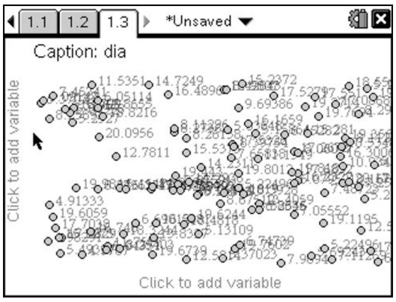
Now we went back to the previous page ((ctrl) ◄) and grabbed a vertex and changed the square in many ways.
We then moved back to the spreadsheet ((ctrl) ►) and students were amazed to see all the captured data looking something like this.
Scrolling down we saw that many items of data had been captured but the magic for many students was that the perimeter divided by the diagonal was always 2.82843.

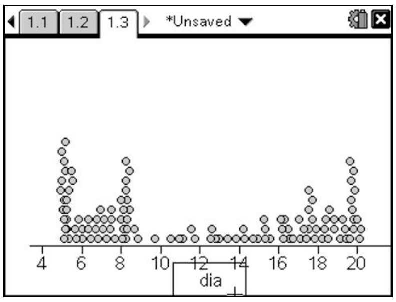


To graph these data we gave the column headings names of *peri* and *dia*.



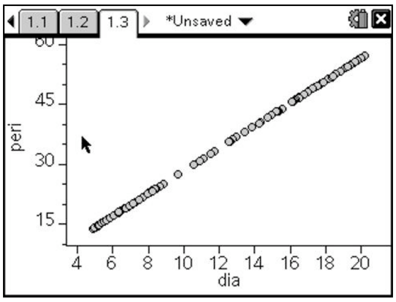
Next we opened a Data & Statistics page: (ctrl) (doc▾) (5).
At first all the data were shown, but totally unordered, so we moved to the bottom of the screen and clicked on the box selecting the variable *dia*.

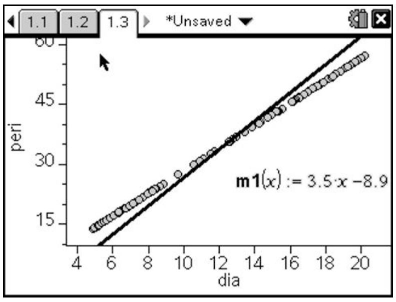




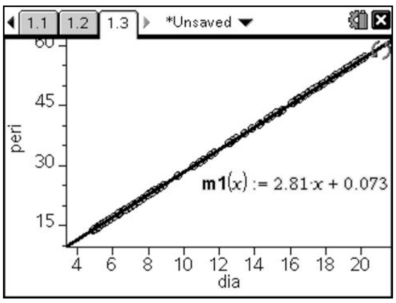
Next we clicked on *peri* at the left hand side of the screen. Students loved the way the data moved into a straight line!

We then added a movable line: (menu) (4) (2).





This could be manipulated to fit over the data by grabbing near one end to rotate it, then gabbing near the middle to translate it into place.
I would like to report that we then talked about the gradient and how this was the same as the perimeter divided by the diagonal, but there just wasn't time to do this in the first lesson! Also the students did not have time to do the Data & Statistics page on their handhelds, but they did observe it on the IWB.



The second lesson

The first lesson had taken a lot of time because there were many new skills that students had to learn and it was important that they felt secure in using all these on the handhelds. The next lesson went far faster since students remembered what to do, or could help others.

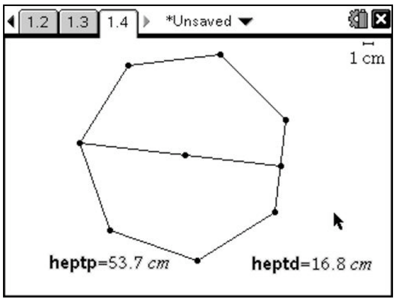
This lesson started in a similar way to the first one, but students now worked with a regular hexagon, then octagon, decagon, dodecagon and some managed to deal with the 14 and 16 sided regular polygon. The following results were obtained:

Number of sides	Perimeter ÷ diameter
4	2.82843
6	3
8	3.06147
10	3.09017
12	3.10583
14	3.11529
16	3.12145

As a plenary we discussed what we noticed about these values. There were comments such as ‘they increase’, ‘the increase is getting smaller’, ‘why is one result exact’ (there hangs another story!), ‘will it increase for ever?’ and ‘what happens if we use an odd number of sides?’.

We decided that we could deal with the diameter of an odd numbered regular polygon by joining a vertex to the mid-point of the opposite side– a diameter must go through the centre

To construct the midpoint of a side, press (menu) [A] (5) and move the cursor to one of the sides. Press (enter). Here is the screen shot for a heptagon.
Now with the regular polygons of odd numbered sides we were able to start with 3 and go up to 15:



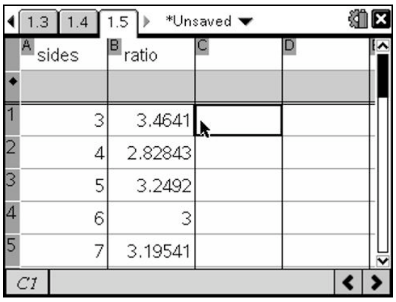
Number of sides	Perimeter ÷ diagonal
3	3.4641
5	3.2492
7	3.19541
9	3.17389
11	3.16312
13	3.15697
15	3.15313

Of course if every student had to work out all these values it would become quite tedious, they worked work in pairs and at the end of the lesson we collected the values together to check results and allow those who were less slick on the button pressing to use the class results.

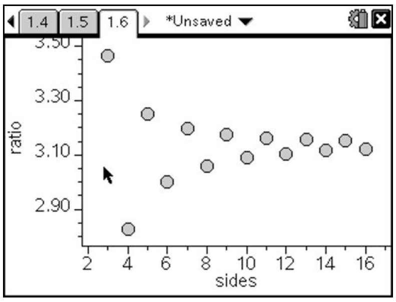
The third lesson

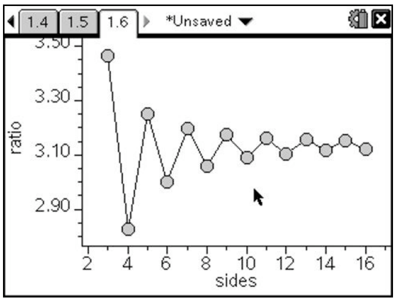
In the final lesson we looked at how to plot both tables as coordinates on a graph, which was fist done on graph paper. We noticed that the sequence oscillated as the number of sides increased but it was settling down to something between 3.15313 and 3.12145.

Then on the TI-Nspire we combined all these data into one spreadsheet and then plotted the points as a scatter graph. We discussed why, strictly, we should not join the points, though we decided that to do so does make it easier for our eye to follow



	A sides	B ratio	C	D
1	3	3.4641		
2	4	2.82843		
3	5	3.2492		
4	6	3		
5	7	3.19541		





In the final plenary it was possible to discuss what the regular polygons would look like if the number of sides were increased without limit. Of course the ratio of perimeter to diameter tends towards π and I was able to use these lessons as an introduction to Circumference ÷ diameter = π, and on to $C = \pi d$.

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